Electron Plasma Waves:-

Eq. of motion is

$$mn\frac{\partial v_e}{\partial t} = -neE - \nabla P_e \quad \text{---(1)}$$

$\because P_e = \gamma K_B T e n$

$\gamma = \frac{N+2}{N} = \frac{1+2}{1} = 3$

$\therefore \nabla P_e = 3K_B T_e \nabla n \quad \text{---(2)}$

linearize this equation (for electron)

$$mn_0\frac{\partial v_1}{\partial t} = -n_0 eE_1 - \nabla P_1 \quad \text{---(3)}$$

$$mn_0\frac{\partial v_1}{\partial t} = -n_0 eE_1 - 3K_B T \nabla n_1 \quad \text{---(4)}$$

$P_1 = 3K_B T_e \nabla n_1$

Equation of continuity

$$\frac{\partial n}{\partial t} + \bar{\nabla}.(nv) = 0$$

linearizing above equation,

$$\frac{\partial n_1}{\partial t} + n_0 \nabla . v_1 = 0 \quad \text{---(5)}$$

Poisson's Equation:

$$\bar{\nabla}.E_1 = -4\pi e n_1 \quad \text{---(6)}$$

Taking the divergence of eq. (4)

$$mn_0\frac{\partial}{\partial t}(\bar{\nabla}.\bar{v}_1) = -n_0 e\bar{\nabla}.\bar{E}_1 - 3K_B T \nabla.\nabla n_1$$

From eq. (6) we have

$$mn_0\frac{\partial}{\partial t}\bar{\nabla}.\bar{v}_1 = -n_0 e(-4\pi e n_1) - 3K_B T \nabla^2 n_1 \quad \text{---(7)}$$

From eq. (5)

$$n_0(\bar{\nabla}.v_1) = -\frac{\partial n_1}{\partial t}$$

$$\bar{\nabla}.v_1 = -\frac{1}{n_0}\frac{\partial n_1}{\partial t}$$

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$$-m\frac{\partial^2}{\partial t^2} n_1 e^{i(kx-\omega t)} = n_0 e^2 4\pi n_1 e^{i(kx-\omega t)} - 3K_B T \frac{\partial^2}{\partial x^2} n_1 e^{i(kx-\omega t)} \cdots$$

$$-\frac{\partial^2}{\partial t^2} n_1 e^{i(kx-\omega t)} = \frac{4\pi n_0 e^2 n_1 e^{i(kx-\omega t)}}{m} - \frac{3K_B T}{m}\frac{\partial^2}{\partial x^2} n_1 e^{i(kx-\omega t)}$$

$$-\frac{\partial^2}{\partial t^2} n_1 = \frac{4\pi n_0 e^2 n_1}{m} - \frac{3K_B T}{m}\nabla^2 n_1$$

$$-\frac{\partial^2 n_1}{\partial t^2} = \omega_{pe}^2 n_1 - \frac{3}{2} V_{th}^2 \nabla^2 n_1 \qquad \because V_{th}^2 = \frac{2k_B T}{m}$$

$$-\frac{\partial^2}{\partial t^2}\left(n_1 e^{i(kx-\omega t)}\right) = \omega_{pe}^2 n_1 e^{i(kx-\omega t)} - \frac{3}{2} V_{th}^2 \frac{\partial^2}{\partial x^2} n_1 e^{i(kx-\omega t)}$$

$$\omega^2 n_1 e^{i(kx-\omega t)} = \omega_{pe}^2 n_1 e^{i(kx-\omega t)} + \frac{3}{2} k^2 V_{th}^2 n_1 e^{i(kx-\omega t)}$$

$$\omega^2 = \omega_{pe}^2 + \frac{3}{2} k^2 V_{th}^2$$

This is the dispersion relation for electron plasma waves. Electron plasma waves are propagating due to temperature effect.

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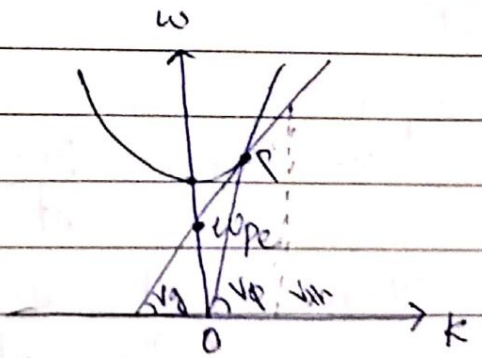
$$\omega^2 - \frac{3}{2} k^2 V_{th}^2 = \omega_{pe}^2 \qquad (*)$$

$$\because \omega_{pe}^2 = \frac{4\pi n_0 e^2}{m_e}$$

$$\frac{\omega^2}{\omega_{pe}^2} - \frac{3}{2}\frac{k^2 V_{th}^2}{\omega_{pe}^2} = 1 \qquad \because V_{th}^2 = \frac{2kT_e}{m}$$

$$\frac{\omega^2}{\omega_{pe}^2} - \frac{k^2}{\omega_{pe}^2/\frac{3}{2}V_{th}^2} = 1$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

From eq. (*)

$$\frac{3}{2} k^2 V_{th}^2 = \omega^2 - \omega_p^2$$

$$k^2 = \frac{2}{3}\frac{\omega^2 - \omega_p^2}{V_{th}^2}$$

Cases:-

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1) If $\omega = \omega_{pe}$ then $k = 0$
It means wave will not propagate.

ii) If $\omega > \omega_{pe}$ then "k" is real.
It means "k" is propagating in +ve x-axis.

iii) If $\omega < \omega_{pe}$ then "k" is imaginary or -ve.

Phase Velocity of Electron Plasma wave:-

$$\omega^2 = \omega_{pe}^2 + \frac{3}{2} k^2 V_{th}^2 \quad \text{---(8)}$$

$$k = \sqrt{\frac{2}{3}} \sqrt{\frac{\omega^2 - \omega_{pe}^2}{V_{th}^2}}$$

$$k = \sqrt{\frac{2}{3}} \omega \sqrt{\frac{1 - \omega_{pe}^2/\omega^2}{V_{th}^2}}$$

$$\frac{k}{\omega} = \sqrt{\frac{2}{3}} \sqrt{\frac{1 - \omega_p^2/\omega^2}{V_{th}^2}}$$

$$\text{Phase Velocity} = V_\phi = \frac{\omega}{k} = \frac{\sqrt{3/2}\, V_{th}}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}}$$

Group Velocity: $V = V_g$

Differentiating eq. (8)

$$2\omega d\omega = 3 \cdot 2k\, dk\, V_{th}^2$$

$$\frac{d\omega}{dk} = \frac{3/2\, 2 V_{th}^2}{\omega/k}$$

$$\frac{d\omega}{dk} = \frac{3/2\, V_{th}^2}{V_\phi}$$

=> Effect of "V_{th}" increases as "k" increases. Pg #89.
Ion sound wave: Pg #97, 1st & 2nd Paragraph. (1st Para)

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Difference b/w Electron Plasma waves & ion waves:

Electron waves	Ion waves
1) are constant frequency waves.	1. are constant velocity waves.
2. v_g & v_ϕ are different.	2. v_g, v_ϕ are same.
3. exist with or without temperature effect.	3. Do not exist without temperature effect.